P5.3

A Non-local Second Moment Closure Model Applied to Convective Boundary Layers

Carol Anne Clayson
Department of Meteorology and Geophysical Fluid Dynamics Institute,
Florida State University, Tallahassee, FL

Lakshmi Kantha
Department of Aerospace Engineering Sciences
University of Colorado, Boulder, CO

Sandro Carniel
Institute for Marine Research, Venice, Italy

We present a non-local second moment closure model for modeling turbulent mixing in the oceanic mixed layer and the atmospheric PBL. The model uses a prognostic equation for the turbulence length scale and incorporates counter-gradient terms for scalar fluxes to better simulate mixing under convective conditions. The results are tested against LES data on convective mixing. We conclude that while the presence of counter-gradient terms in scalar fluxes is a highly desirable feature, it is necessary to reexamine the modeling of third moments.

1. Introduction

Second moment closure is being increasingly invoked to model turbulent mixing. The most common form is a two-equation turbulence model where prognostic equations are solved for the turbulence kinetic energy as well as a quantity involving the length scale, while algebraic relations are written down for second moments. Examples are the Mellor-Yamada type q^2-q^2l models (Mellor and Yamada 1982, Kantha and Clayson 1994) in geophysical applications to model turbulent mixing in the atmospheric boundary layer and the oceanic mixed layer, and *k-ε* models in engineering (Rodi 1989). For an excellent review of recent developments in oceanic mixed layer modeling (up to 2001), see Burchard (2002).

However, a perceived shortcoming of these models is the down-the-gradient approximation for turbulent mixing that is not accurate when applied to convective boundary layers. For example, the heat flux (kinematic) is modeled in Mellor and Yamada (1982, MY henceforth) and Kantha and Clayson (1994, KC henceforth) models as:

$$\overline{w\theta} = -q\ell S_H \frac{\partial \Theta}{\partial z} \qquad (1)$$

where q is the turbulence velocity, ℓ is the turbulence length scale and S_H is the stability function and Θ is the temperature. There is ample evidence to suggest that under convective conditions, this equation should be modified by a counter-gradient term (Deardorff 1972, Large et al. 1994, see also Cheng et al. 2002):

$$\overline{w\theta} = -q\ell S_H \left(\frac{\partial \Theta}{\partial z} - \gamma_c^\theta \right) \qquad (2)$$

Closure at the second moment level consists of writing conservation equations for second moments of turbulence quantities and modeling the higher order terms in these equations (for details see Mellor and Yamada 1982, Cheng et al. 2002). The full set consisting of differential equations for six Reynolds stresses $-\rho\overline{u_i u_j}$, three turbulent scalar fluxes $-\rho c_p \overline{u_i \theta}$ and scalar variance $\overline{\theta^2}$ is nearly impractical for inclusion in large three-dimensional models. Consequently, Mellor and Yamada (1974) employed simple perturbation expansion scheme to derive a hierarchy of increasingly simple closure models. The first one is the Level 3 model, in which the turbulent stresses and scalar fluxes are approximated by algebraic equations obtained by neglecting the material derivative and diffusion terms. This model consists of algebraic equations for turbulence quantities $\overline{u_i u_j}$ and $\overline{u_i \theta}$ and differential equations for q^2 and $\overline{\theta^2}$. Mellor and Yamada (1974) also obtained a Level 21/2 model in which an algebraic equation is derived instead for $\overline{\theta^2}$ by arbitrarily neglecting the material derivative and diffusion terms in the differential equation for $\overline{\theta^2}$.

A great advantage of the Mellor and Yamada (1982) Level 21/2 model and its variants (see Galperin et al. 1988, Kantha and Clayson 1994, Kantha 2003, see also Cheng et al. 2002) is the ready applicability to the ocean and the atmosphere, where the density of the fluid parcel is determined by more than one property (temperature and salinity in the ocean; temperature, water vapor and liquid water in the atmosphere). The Level 3 model requires that differential equations be written for the variances of these quantities as well as the cross correlations between them. Level 21/2 avoids this complication and hence has been quite popular in geophysical applications.

However, while neglecting the material derivative and diffusion terms in the scalar variance equations is tolerable for stably stratified flows, the approximation becomes less accurate under unstable stratification with the largest error occurring at the free convection limit. The atmospheric boundary layer is mostly convection-driven and under certain conditions such as during nocturnal and winter cooling, convection dominates in the ocean also. In free convection, there is a significant counter-gradient scalar flux. The Level 21/2 model treats turbulent mixing as entirely a down-the-gradient process. This has been regarded as its major flaw and has led some researchers to construct simple nonlocal models (Large et al. 1994 for example) to account for the counter-gradient scalar fluxes under convective conditions.

Nakanishi (2001) and Cheng et al. (2002) point out that the Level 3 model in the Mellor-Yamada hierarchy carries the advantage that it has the capability to produce counter-gradient scalar fluxes. This is definitely a desirable property for application to the convective PBL. While the Level 21/2 model reproduces the convective PBL depth reasonably well (it does underestimate it somewhat), it does not do a good job in reproducing the distribution of turbulence quantities in the convectively-mixed layer. Nakanishi (2001) developed and compared a Level 3 model for the PBL with LES simulations, and Nakanishi and Niino (2004) have applied it to the PBL with radiation fog. However, both these studies use a diagnostic equation for the turbulence length scale, as is the usual practice in the atmospheric PBL community. In this paper, we develop a Level 3, non-local model with a prognostic equation for the length scale for applications primarily to the oceanic mixed layer, but potentially to the atmospheric PBL as well.

2. Level 3 Non-local Model and Counter-gradient Terms

Nakanishi (2001) and Nakanishi and Niino (2004) derive the Level 3 model following the original Mellor and Yamada (1974) expansion scheme. Here we follow a similar approach but for the case of a quasi-equilibrium model, which employs a slightly modified expansion scheme and has better numerical stability (Galperin et al. 1988, Kantha and Clayson 1994, Kantha 2003). The notation is however that of Cheng et al. (2002).

Without loss of generality, one can orient the x-axis in the direction of the mean flow so that the algebraic relations for the Reynolds stresses and the turbulent heat fluxes (because of the neglect of tendency and diffusion terms) become:

$$
\begin{aligned}
\overline{u^2} &= \frac{q^2}{3}\left(1-2\lambda_4\right) - \frac{\tau}{3}\left[\left(\lambda_2+3\lambda_3+2\lambda_4\right)\overline{uw}\frac{\partial U}{\partial z}\right] \\
\overline{v^2} &= \frac{q^2}{3}\left(1-2\lambda_4\right) - \frac{\tau}{3}\left[-2\left(\lambda_2-\lambda_4\right)\overline{uw}\frac{\partial U}{\partial z}\right] \\
\overline{w^2} &= \frac{q^2}{3}\left(1-3\lambda_3+\lambda_2\right) + \frac{\tau}{3}\left[\left(-\lambda_2+3\lambda_3+4\lambda_4\right)g\alpha\overline{w\theta}\right] \\
\overline{uv} &= 0 \\
\overline{uw} &= -\frac{\tau}{2}\frac{\partial U}{\partial z}\left[\frac{1}{2}\left(\lambda_1-\frac{4}{3}\lambda_2\right)q^2 + \left(\lambda_2-\lambda_3\right)\overline{u^2} + \left(\lambda_2+\lambda_3\right)\overline{w^2}\right] \\
&\quad + \lambda_4\tau g\alpha\overline{u\theta} \\
\overline{vw} &= 0 \\
\overline{u\theta} &= -\frac{\tau}{\lambda_5}\left[\overline{uw}\frac{\partial\Theta}{\partial z} + \frac{1}{2}\left(\lambda_6+\lambda_7\right)\overline{w\theta}\frac{\partial U}{\partial z}\right] \\
\overline{v\theta} &= 0 \\
\overline{w\theta} &= -\frac{\tau}{\lambda_5}\left[\overline{w^2}\frac{\partial\Theta}{\partial z} + \frac{1}{2}\left(\lambda_6-\lambda_7\right)\overline{u\theta}\frac{\partial U}{\partial z} - \lambda_0 g\alpha\overline{\theta^2}\right]
\end{aligned}
\tag{3}
$$

where $\tau = B_1\,\ell/q$ is the turbulence time scale; B_1 and λ_i (i = 0, 1, … 8) are closure constants. U is the mean velocity and Θ is the mean temperature. α is the expansion coefficient and g is the gravitational acceleration. We ignore salinity (water vapor and liquid water in the atmosphere) for the time being.

Note the appearance of $\overline{\theta^2}$ in the last equation of the set, which in a Level 3 model, is governed by the differential equation:

$$
\frac{D\left(\overline{\theta^2}\right)}{Dt} + \frac{\partial}{\partial z}\left(\overline{w\theta^2}\right) = -2\overline{w\theta}\frac{\partial\Theta}{\partial z} - 2\frac{\overline{\theta^2}}{\tau_\theta} \tag{4}
$$

where $\tau_\theta = B_2\,\ell/q$ is another turbulence time scale. Note that in Level 21/2 model of the Mellor-Yamada hierarchy, the advection and diffusion terms on the LHS of Eq. (4) are neglected without any rigorous justification so that an algebraic equation can also be written for $\overline{\theta^2}$. This enables $\overline{\theta^2}$ in the last equation of the set (3) to be replaced by $\overline{w\theta}$. However, this also means that counter-gradient terms have to be omitted from the expression for the scalar fluxes so that, the kinematic heat flux $\overline{w\theta}$ is represented by Eq. (1) and not Eq. (2). If we now write:

$$
\overline{uw} = -\frac{q^2}{2}\tau S_{MC}\frac{\partial U}{\partial z};\quad \overline{w\theta} = -\frac{q^2}{2}\tau S_{HC}\frac{\partial\Theta}{\partial z} \tag{6}
$$

where S_{MC} and S_{HC} are stability functions. Subscript C denotes quantities as defined by Cheng et al. (2002).

If we put $S_{MC} = S'_{MC} + S''_{MC}$ and $S_{HC} = S'_{HC} + S''_{HC}$, where primes denote the Level 2.5 model, and double primes the counter-gradient terms, it can be shown that

$$
\begin{aligned}
S'_{MC} &= \frac{1}{D}\left(s_0 + s_1 G_{HC} + s_2 G_{MC}\right) \\
S'_{HC} &= \frac{1}{D}\left(s_4 + s_5 G_{HC} + s_6 G_{MC}\right) \\
D &= 1 + d_1 G_{HC} + d_2 G_{MC} + d_3 G_{HC}^2 + d_4 G_{HC} G_{MC} + d_5 G_{MC}^2 \\
S''_{MC} &= \frac{1}{\overline{D}}\left(s_7 G_{HC}^2\right)\left(C_\theta - \frac{\lambda_8}{2\lambda_0} S'_{HC}\right) \\
S''_{HC} &= \frac{1}{\overline{D}}\left(s_8 + s_9 G_{HC}^2 + s_{10} G_{MC} G_{HC}\right)\left(C_\theta - \frac{\lambda_8}{2\lambda_0} S'_{HC}\right) \\
\overline{D} &= 1 + \overline{d}_1 G_{HC} + \overline{d}_2 G_{MC} + \overline{d}_3 G_{HC}^2 + \overline{d}_4 G_{HC} G_{MC} + \overline{d}_5 G_{MC}^2
\end{aligned} \tag{7}
$$

where

$$
\begin{aligned}
G_{MC} &= (\tau S)^2 ; S^2 = \left(\frac{\partial U}{\partial z}\right)^2 \\
G_{HC} &= (\tau N)^2 ; N^2 = g\alpha\left(\frac{\partial \Theta}{\partial z}\right)
\end{aligned} \tag{8}
$$

and

$$
\begin{aligned}
d_1 &= \frac{1}{\lambda_5}\left(-\frac{\lambda_2}{3} + \lambda_3 + \frac{7}{3}\lambda_4 + \lambda_8\right) \\
d_2 &= -\frac{1}{6}(\lambda_2 - \lambda_3)(\lambda_2 + 3\lambda_3 + 2\lambda_4) - \frac{\lambda_6^2 - \lambda_7^2}{4\lambda_5^2} \\
d_3 &= \frac{\lambda_4}{3\lambda_5^2}(-\lambda_2 + 3\lambda_3 + 4\lambda_4 + 3\lambda_8) \\
d_4 &= \frac{1}{12\lambda_5^2}(\lambda_2 + \lambda_3)(\lambda_6 - \lambda_7)(-\lambda_2 + 3\lambda_3 + 4\lambda_4) \\
&\quad - \frac{1}{18\lambda_5}(\lambda_2 - \lambda_3)(\lambda_2 + 3\lambda_3 + 2\lambda_4)(-\lambda_2 + 3\lambda_3 + 4\lambda_4) \\
&\quad - \frac{\lambda_8}{6\lambda_5}(\lambda_2 - \lambda_3)(\lambda_2 + 3\lambda_3 + 2\lambda_4) \\
d_5 &= \frac{1}{24\lambda_5^2}(\lambda_2 - \lambda_3)(\lambda_6^2 - \lambda_7^2)(\lambda_2 + 3\lambda_3 + 2\lambda_4)
\end{aligned} \tag{9}
$$

The overbarred quantities $\overline{d}_i, i = 1,5$ can be obtained from Eq. (9) by putting $\lambda_8 = 0$ in the corresponding expressions for d_i.

$$
\begin{aligned}
s_0 &= \frac{2}{3\lambda_5}(1 + \lambda_2 - 3\lambda_3) \\
s_1 &= \frac{2\lambda_4}{3\lambda_5^2}(1 + \lambda_2 - 3\lambda_3) \\
s_2 &= \frac{1}{12\lambda_5^2}(\lambda_6 - \lambda_7)\begin{bmatrix}(3\lambda_1 - 4\lambda_2) + 2(\lambda_2 - \lambda_3)(1 - 2\lambda_4) \\ +2(\lambda_2 + \lambda_3)(1 - 2\lambda_4)(1 + \lambda_2 - 3\lambda_3)\end{bmatrix} \\
&\quad - \frac{1}{9\lambda_5}(1 - \lambda_2 - 3\lambda_3)(\lambda_2 - \lambda_3)(\lambda_2 + 3\lambda_3 + 2\lambda_4) \\
s_4 &= \frac{1}{6}\left[(3\lambda_1 - 4\lambda_2) + 2(\lambda_2 - \lambda_3)(1 - 2\lambda_4) + 2(\lambda_2 + \lambda_3)(1 + \lambda_2 - 3\lambda_3)\right] \\
s_5 &= \frac{1}{18\lambda_5}(-\lambda_2 + 3\lambda_3 + 4\lambda_4 + 3\lambda_8)\begin{bmatrix}(3\lambda_1 - 4\lambda_2) + 2(\lambda_2 - \lambda_3)(1 - 2\lambda_4) \\ +2(\lambda_2 + \lambda_3)(1 + \lambda_2 - 3\lambda_3)\end{bmatrix} \\
&\quad - \frac{1}{9\lambda_5^2}(1 + \lambda_2 - 3\lambda_3)\left[3\lambda_4(\lambda_6 + \lambda_7) + \lambda_5(\lambda_2 + \lambda_3)(-\lambda_2 + 3\lambda_3 + 4\lambda_4)\right] \\
s_6 &= -\frac{1}{24\lambda_5^2}(\lambda_6^2 - \lambda_7^2)\begin{bmatrix}(3\lambda_1 - 4\lambda_2) + 2(\lambda_2 - \lambda_3)(1 - 2\lambda_4) \\ +2(\lambda_2 + \lambda_3)(1 + \lambda_2 - 3\lambda_3)\end{bmatrix} \\
s_7 &= \frac{\lambda_0}{\lambda_5^2}\left[\lambda_4(\lambda_6 + \lambda_7) + \frac{\lambda_5}{3}(\lambda_2 + \lambda_3)(-\lambda_2 + 3\lambda_3 + 4\lambda_4)\right] \\
s_8 &= -\frac{2\lambda_0}{\lambda_5}; s_9 = -\frac{2\lambda_0\lambda_4}{\lambda_5^2} \\
s_{10} &= \frac{\lambda_0}{3\lambda_5}\left[(\lambda_2 - \lambda_3)(\lambda_2 + 3\lambda_3 + 2\lambda_4)\right]
\end{aligned} \tag{10}
$$

Note that the first three equations in set (3) differ from those of Cheng et al. (2002):

$$
\begin{aligned}
\overline{u^2} &= \frac{q^2}{3} - \frac{\tau}{3}\left[(\lambda_2 + 3\lambda_3)\overline{uw}\frac{\partial U}{\partial z} + 2\lambda_4 g\alpha\overline{w\theta}\right] \\
\overline{v^2} &= \frac{q^2}{3} - \frac{\tau}{3}\left[-2\lambda_2\overline{uw}\frac{\partial U}{\partial z} + 2\lambda_4 g\alpha\overline{w\theta}\right] \\
\overline{w^2} &= \frac{q^2}{3} + \frac{\tau}{3}\left[(-\lambda_2 + 3\lambda_3)\overline{uw}\frac{\partial U}{\partial z} + 4\lambda_4 g\alpha\overline{w\theta}\right]
\end{aligned} \tag{11}
$$

The corresponding values for the non-equilibrium Level 2.5 model can be found in Cheng et al. (2002, see also Kantha 2003) except

$$
\begin{aligned}
s_7 &= \frac{\lambda_0}{\lambda_5^2}\left[\lambda_4(\lambda_6 + \lambda_7) + \frac{2\lambda_4\lambda_5}{3}(\lambda_2 + 3\lambda_3)\right] \\
s_8 &= -\frac{2\lambda_0}{\lambda_5}; s_9 = -\frac{2\lambda_0\lambda_4}{\lambda_5^2} \\
s_{10} &= \frac{2\lambda_0}{3\lambda_5}(\lambda_2^2 - 3\lambda_3^2)
\end{aligned} \tag{12}
$$

The Level 3 equivalent of the KC model (in their notation) can be found by putting (see Kantha 2003) $\lambda_0 = (1\text{-}C_3)$, $\lambda_1 = 4B_1^{-4/3} + 24(A_1/B_1)^2$, $\lambda_2 = \lambda_3 = \lambda_4 = 3A_1/B_1$, $\lambda_5 = B_1/(3A_2)$, $\lambda_6 = \lambda_7 = (1\text{-}C_2)$ and $\lambda_8 = B_2(1\text{-}C_3)/B_1$. This yields:

$$S'_H = \frac{A_2\left[1-\left(6A_1/B_1\right)\right]}{1-3A_2\left[6A_1+B_2\left(1-C_3\right)\right]G_H}$$

$$S'_M = A_1\left\{\frac{\left[1-(6A_1/B_1)-3C_1\right]+9\left[2A_1+A_2(1-C_2)\right]S'_H G_H}{\left(1-9A_1A_2G_H\right)}\right\} \quad (13)$$

$$S''_H = \left\{\frac{3A_2(1-C_3)G_H}{\left(1-18A_1A_2G_H\right)}\right\}\left(C_\theta - B_2 S'_H\right)$$

$$S''_M = \left\{\frac{9A_1\left[2A_1+A_2(1-C_2)\right]G_H}{(1-9A_1A_2G_H)}\right\}S''_H$$

where $C_\theta = \dfrac{\overline{\theta^2}}{\ell^2\Theta_z^2}$ and we have made use of the identities:

$$S_M = \frac{B_1}{2}S_{MC}, S_H = \frac{B_1}{2}S_{HC}$$
$$G_M = -B_1^2 G_{MC}, G_H = -B_1^2 G_{HC} \quad (14)$$

Note that the expressions for S'_M, S'_H are the same as those for the Level 2 1/2 model (Kantha and Clayson 1994, Kantha 2003). The values of the closure constants are (Kantha 2003): $A_1 = 0.58$, $B_1 = 16.6$, $C_1 = 0.0384$, $A_2 = 0.62$, $B_2 = 12.04$, $C_2 = 0.429$ and $C_3 = 0.2$.

Note also that Eq. (2) implies: $\gamma_c^\theta = -\left(\dfrac{S''_H}{S'_H}\right)\Theta_z$ (15)

3. Application to Oceanic Mixing

In applications to oceanic (atmospheric) mixing, we must include salinity (water vapor and liquid water). The equation set (3) becomes (in KC notation):

$$\overline{u^2} = \frac{q^2}{3}\left(1-\frac{6A_1}{B_1}\right) - \frac{6A_1\ell}{q}\left(\overline{uw}\frac{\partial U}{\partial z}\right)$$
$$\overline{v^2} = \frac{q^2}{3}\left(1-\frac{6A_1}{B_1}\right)$$
$$\overline{w^2} = \frac{q^2}{3}\left(1-\frac{6A_1}{B_1}\right) + \frac{6A_1\ell}{q}\left[g\alpha\overline{w\theta} + g\beta\overline{ws}\right]$$
$$\overline{uv} = 0$$
$$\overline{uw} = -\frac{3A_1\ell}{q}\left[\left(\overline{w^2} - C_1 q^2\right)\frac{\partial U}{\partial z} - \left(g\alpha\overline{u\theta} + g\beta\overline{us}\right)\right]$$
$$\overline{vw} = 0 \quad (16)$$
$$\overline{u\theta} = -\frac{3A_2\ell}{q}\left[\overline{uw}\frac{\partial\Theta}{\partial z} + \left(1-C_2\right)\overline{w\theta}\frac{\partial U}{\partial z}\right]$$
$$\overline{us} = -\frac{3A_2\ell}{q}\left[\overline{uw}\frac{\partial S}{\partial z} + \left(1-C_2\right)\overline{ws}\frac{\partial U}{\partial z}\right]$$
$$\overline{v\theta} = 0;\ \ \overline{vs} = 0$$
$$\overline{w\theta} = -\frac{3A_2\ell}{q}\left[\overline{w^2}\frac{\partial\Theta}{\partial z} - \lambda_0 g\alpha\overline{\theta^2} - \lambda_0 g\beta\overline{\theta s}\right]$$
$$\overline{ws} = -\frac{3A_2\ell}{q}\left[\overline{w^2}\frac{\partial S}{\partial z} - \lambda_0 g\beta\overline{s^2} - \lambda_0 g\alpha\overline{\theta s}\right]$$

Additional differential equations must also be written down for the temperature and salinity variances, and the correlation between temperature and salinity:

$$\frac{D\left(\overline{\theta^2}\right)}{Dt} - \frac{\partial}{\partial z}\left(q\ell S_\theta\frac{\partial\left(\overline{\theta^2}\right)}{\partial z}\right) = -2\overline{w\theta}\frac{\partial\Theta}{\partial z} - 2\frac{\overline{\theta^2}}{\tau_\theta}$$
$$\frac{D\left(\overline{s^2}\right)}{Dt} - \frac{\partial}{\partial z}\left(q\ell S_s\frac{\partial\left(\overline{s^2}\right)}{\partial z}\right) = -2\overline{ws}\frac{\partial S}{\partial z} - 2\frac{\overline{s^2}}{\tau_s} \quad (17)$$
$$\frac{D\left(\overline{\theta s}\right)}{Dt} - \frac{\partial}{\partial z}\left(q\ell S_{\theta s}\frac{\partial\left(\overline{\theta s}\right)}{\partial z}\right) = -\overline{ws}\frac{\partial\Theta}{\partial z} - \overline{w\theta}\frac{\partial S}{\partial z} - \varepsilon_{\theta s}$$

It is reasonable to assume $S_\theta = S_s = S_{\theta s}$ and $\tau_\theta = \tau_S = B_2\ell/q$ since these are all scalars. In addition, just as is done in second moment closure for the cross correlations of velocity and temperature (Mellor and Yamada 1982), it is reasonable to assume that the dissipation rate of the cross correlation between temperature and salinity is zero. If we neglect the material derivative and the diffusion terms also, then the production term must also vanish and one gets $\overline{\theta s} = 0$. This is a significant simplification in the analysis.

In addition differential equations for q^2 (twice the turbulence kinetic energy) and for a quantity containing the length scale ($q^2\ell$ in the MY type models) are needed:

$$\frac{D\left(q^2\right)}{Dt} - \frac{\partial}{\partial z}\left(S_q q\ell\frac{\partial\left(q^2\right)}{\partial z}\right) = 2\left(P + B - \varepsilon\right)$$
$$\frac{D\left(q^2\ell\right)}{Dt} - \frac{\partial}{\partial z}\left(S_\ell q\ell\frac{\partial\left(q^2\ell\right)}{\partial z}\right) = \frac{\left(q^2\ell\right)}{q^2}\left(E_1P + E_3B - E_2\varepsilon\zeta\right) \quad (18)$$

where $P = -\overline{uw}\dfrac{\partial U}{\partial z} - \overline{vw}\dfrac{\partial V}{\partial z} = q\ell S_M\left[\left(\dfrac{\partial U}{\partial z}\right)^2 + \left(\dfrac{\partial V}{\partial z}\right)^2\right]$ is the rate of production by shear, $\varepsilon = \dfrac{q^3}{B_1\ell}$ is the dissipation rate and $B = g\alpha\overline{w\theta} + g\beta\overline{ws} = -q\ell S_H\left(g\alpha\dfrac{\partial\Theta}{\partial z} + g\beta\dfrac{\partial S}{\partial z}\right)$ is the rate of production/destruction by buoyancy of the TKE. $\zeta = 1 + E_5\left(\dfrac{\ell}{\kappa\ell_w}\right)^2$ is the wall function needed to keep the turbulence diffusivity coefficient S_ℓ positive definite in the logarithmic region of a turbulent boundary layer; ℓ_w is the distance from the wall and κ is the von Karman constant.

MY and KC chose $E_1 = E_3 = 1.8, E_2 = 1, E_5 = 1.33$. $E_3 = 5.0$ under stable stratification to limit the length scale. $E_5 = 4.8$ to simulate surface wave breaking effects more accurately (Kantha and Clayson 2004).

This then constitutes a four-equation second moment closure-based nonlocal model of turbulence. The four equations to be solved are for 4 quantities: q^2, $q^2 \ell$ (Eq. 18) and $\overline{\theta^2}, \overline{s^2}$ (Eq. 17). Thus we have added 2 additional equations to include counter-gradient terms absent in the Level 2 1/2 model. We will evaluate the performance of this model by comparison with LES data on laboratory convection experiments. Note that the counter-gradient terms are assumed to be zero when the stratification is stable.

4. Application to Laboratory Experiments on Convection

Willis and Deardorff (1974) and Kantha (1980) performed experiments on the deepening of convective mixed layers. Kantha (1980) used a two-layer stably stratified fluid with salt flux at the top to drive convection and measured the deepening rate of the mixed layer. He observed that the ratio of the buoyancy flux at the entraining buoyancy interface to the imposed buoyancy flux Q_b/Q_0 clusters around –0.2, but can vary within the range 0 to 1, depending on the stability of the entraining interface as indicated by a bulk Richardson number. The advantage of this setup is that the entrainment rate remains constant as long as the buoyancy flux is kept constant. However, these experiments are harder to simulate numerically.

Willis and Deardorff (1974) measured the deepening rate of a convective mixed layer in an initially linearly stratified fluid heated from the bottom. In this case, the entrainment rate decreases as time goes on and the buoyancy interface grows progressively stronger. The mixed layer depth is observed to follow $D_m = c\frac{(B_0 t)^{1/2}}{N_0}$, where B_0 is the imposed buoyancy flux and N_0 is the buoyancy frequency of the initial stratification. Mironov et al. (2000) have simulated this experiment numerically by using LES. Their results will be compared with the current model.

Following Burchard (2001), the numerical simulation starts with an initial linear stable stratification of 1 ^{o}C m^{-1} (corresponding to N_0 of 2.56 x 10^{-4} s) with a surface temperature of 22 ^{o}C. A heat loss of 100 W m^{-2} is imposed at the top of the water column corresponding to an imposed buoyancy flux of 0.52 x 10^{-7} m^3 s^{-2}. the salinity is kept constant at 35 psu. The model is integrated for 3 days. At the end of 3 days the mixed layer is roughly 12 m deep, a result that almost all models reproduce reasonably well. The nonpenetrative (zero entrainment at the buoyancy interface) value (c ~ 1.41) for D_m is 11.6 m. So the convective turbulence produces a difference of only about 4%, and this is why it is hard to distinguish between the performances of different models. However, the profiles of temperature and various turbulence parameters obtained from LES simulation are more critical discriminators. The data are normalized by the length scale D_m, the Deardorff convective velocity scale $w_* = (B_0 D_m)^{1/3}$ and the convective temperature scale $\theta_* = \left(\overline{w\theta}\right)_0 / w_*$. We will present the results of Level 21/2 and Level 3 models compared to LES simulations of Mironov et al. (2000).

Mailhot and Benoit (1982) and Large et al. (1994) have used the following expression for the counter-gradient term: $\gamma_c^\theta = c_0 K_H \frac{\left(\overline{w\theta}\right)_0}{w_* D_m}$ (19)

where c_0 ~ 5. This value can be compared with that produced by the Level 3 model. Large et al. (1994) tuned their model to produce Q_b/Q_0 = -0.2 and obtained a mixed layer depth of 13 m at the end of 3 days. Because of the quasi-slab nature of their mixed layer, they obtained a near-uniform temperature in the mixed layer except at the surface and the buoyancy interface.

5. Concluding Remarks

At a first glance, the non-local Level 3 model with counter-gradient terms should be expected to perform better than the local 2 1/2 model. However, while the presence of the counter-gradient terms is certainly desirable under convective conditions, the modeling of the turbulent diffusion terms needs to be refined. This means essentially that the third order moments must be looked at more carefully, as Cheng and Canuto (1994) and Canuto et al. (2001a,b, 2002) have suggested. Canuto et al. (1994) obtained very good agreement with LES data, but their model involved differential equations for $\overline{w^2}$, $\overline{u^2}$, $\overline{w\theta}$, $\overline{\theta^2}$ and ε (and a quasi-normal-closure-based algebraic relationships for third moments). The question is: Can a much simpler model such as the one presented here be made to perform satisfactorily for convective boundary layers?

Acknowledgements. CAC acknowledges the support of the Office of Naval Research under grants N00014-00-1-0611 and N00014-03-1-0989. LK thanks ONR and Dr. Manuel Fiadeiro for support for this work through ONR grant N00014-03-1-0488.

References

Burchard, H., 2002. *Applied Turbulence Modelling in Marine Waters*, Springer, pp 215.

Canuto, V. M., Minotti, F., Ronchi, C., Ypma, R. M., and Zeman, O., 1994. Second-order closure PBL model with new third-order moments: Comparison with LES data. *J. Atmos. Sci.*, 51, 1605-1618.

Canuto, V. M., Cheng, Y., and Howard, A. M., 2001. New third order moments for the convective boundary layer. *J. Atmos. Sci.,* 58, 1069-1172.

Canuto, V. M., Cheng, Y., and Howard, A. M., 2004. What causes the divergences in local second-moment closure models? *J. Atmos. Sci.,* (submitted).

Canuto, V. M., Howard, A. M., Cheng, Y., and Dubovikov, M. S., 2001b. Ocean turbulence. Part I: one-point closure model. Momentum and heat vertical diffusivities. *J. Phys. Oceanogr.*, 31, 1413-1426.

Canuto, V. M., Howard, A. M., Cheng, Y., and Dubovikov, M. S., 2002. Ocean turbulence. Part II: Vertical diffusivities of momentum, heat, salt, mass and passive scalars. *J. Phys. Oceanogr.,* 32, 240-264.

Cheng, Y., and Canuto, V. M., 1994. Stably stratified shear turbulence: a new model for the energy dissipation length scale. *J. Atmos. Sci. 5*1,2384-2396.

Cheng, Y., V. M. Canuto, and A. M. Howard, 2002: An improved model for the turbulent PBL. *J. Atmos. Sci.* 59, 1550-1565.

Deardorff, J. W., 1972. Theoretical expression for the counter-gradient vertical heat flux. *J. Geophys. Res.,* 77, 5900-5904.

Galperin, B., L. H. Kantha, S. Hassid, and A. Rosati, 1988: A quasi-equilibrium turbulent energy model for geophysical flows. *J. Atmos. Sci*., **45**, 55-62.

Kantha, L.H., 1980. Experimental simulation of the 'retreat' of the seasonal thermocline by surface heating. In *Fjord Oceanography*, edited by H.J. Freeland, D.M. Farmer and C.D. Levings, Plenum Press, 197-204.

Kantha, L. H., 2003, On an improved model of the turbulent PBL, *J. Atmos. Sci.* 60, 2239-2246.

Kantha, L. H., 2004. The length scale equation in turbulence models. *Nonlin. Processes Geophys.*, 11, .

Kantha, L. H. and Clayson, C. A. 1994: An improved mixed layer model for geophysical applications. *J. Geophys. Res.* 99, 25235-25266.

Kantha, L. H., and Clayson, C. A. 2004. On the effect of surface gravity waves on mixing in the oceanic mixed layer. *Ocean Modelling ,* 6, 101-124.

Large, W. G., McWilliams, J. C., and Doney, S. C., 1994. A review and model with nonlocal boundary layer parameterization. *Rev. Geophys.* 32, 363-403.

Mailhot, J. and R. Benoit, 1982. A finite element model of the atmospheric boundary layer suitable for use in numerical weather prediction models, *J. Atmos. Sci.,* 39, 2249-2266.

Mellor, G. L. and Yamada, T. 1982. Development of a turbulence closure model for geophysical fluid problems. *Rev. Geophys. Space Phys., 20*, 851-875.

Mironov, D. V., V. M. Gryanik, C. H. Moeng, D. J. Olbers, and T. Warncke, 2000. Vertical turbulence structure and second moment budgets in convection with rotation: A large eddy simulation study. *Q. J. Roy. Meteorol. Soc.,* 126, 477-515.

Nakanishi, M., 2001. Improvement of the Mellor-Yamada turbulence closure model based on large eddy simulation data. *Boundary-Layer Meteor.,* **99**, 349-378.

Nakanishi, M., and H. Niino, 2004. An improved Mellor-Yamada Level-3 model with condensation physics: its design and verification. *Boundary-Layer Meteor.,* **112**, 1-31.

Willis, G. E., and J. W. Deardorff, 1974. A laboratory model of the unstable planetary boundary layer. *J. Atmos. Sci.,* 31, 1297-1307.